Are conjurations catastrophes?*

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*Procession in Valencia. Studio Francisco José de Goya y Lucientes, 1746-1828. Kunsthaus Zurich.

How can uniform opinions diverge in a harmonious society and lead to conspiracy theories? The catastrophe theory offers an answer.

As a result of the crisis related to the COVID-19 epidemic, alternative theories to the official political-scientific explanations were born and spread in our societies at lightning speed. Some of these theories seemed to be imported from external sources with the aim of social or political destabilization. Others, and these are the most interesting, emerged almost spontaneously in predisposed layers of our social basement. In both cases, they showed fractures that were barely perceptible before[1]. Indeed, when a tense situation unfolds in the country, the community enters a polarized state. It is no longer tolerated to present fuzzy opinions, and any change of opinion does only happen in an abrupt manner.

Geometric catastrophes

Does the catastrophe theory developed in the 1960s by the French mathematician René Thom offer an explanation for this phenomenon? His theory does not necessarily consider earthquakes, wars, epidemics and other similar calamities, but seeks to find out how abrupt changes in the state of a system can occur. The core idea of Thom's theory is contained in the title of his major work: "Modèles Mathématiques de la Morphogenèse". Thom asserts in a philosophical manner that differentiated and even harmonic natural shapes are produced by foldings in nature.

The aim of this essay is to prove the plain sailing connection insinuated in its title.

First, we need to classify some terms such as "Sensitive, Rational, Resonance, Response, Threat and their Perception". Often do people develop different perceptions in the face of a threat that is the same for all. Some advocate more cool and rational attitudes. Others, whose sensitivity or sensibility is more pronounced, tend to exhibit distinctive responses or resonances. I deliberately do not want to repeat definitions here, nor do I want to try quantitative assessments, although these would be possible by means of psychological tests. My concern is to give these terms a qualitative, geometric shape.

Under such simple hypotheses, the problem can be represented in a three-dimensional space. Figure 1 first shows the reaction of a single person. The horizontal axis behind describes his attitude, which can vary between sensitivity and cool rationality. The second horizontal axis represents his perceived threat: if the values on this axis are small, then the threat is low; if they are high, it is considered considerable by this person. The vertical axis measures that person's response in accordance to their attitude and perceived threat. If the

[1] Such skirmishes are pursued on other battlefields, one thinks only at the LGTB controversy or at the commotion around inclusive writing.

values on the vertical axis are low, the person responds with low resonance to the threat. In contrast, with higher values, he will demonstrate a will to fight in response. In the jargon of the catastrophe theory, the blue horizontal plane where attitude and threat interact is called the ***control space***. The vertical axis that describes the resonance or response or attitude of our human being is called the ***state space***. Finally, for each coordinate of the control plane, an attitude is mapped onto the state space. As an example, the red dots depict two people, one more sensitive and calm, the other more rational and worried, or eager to fight.

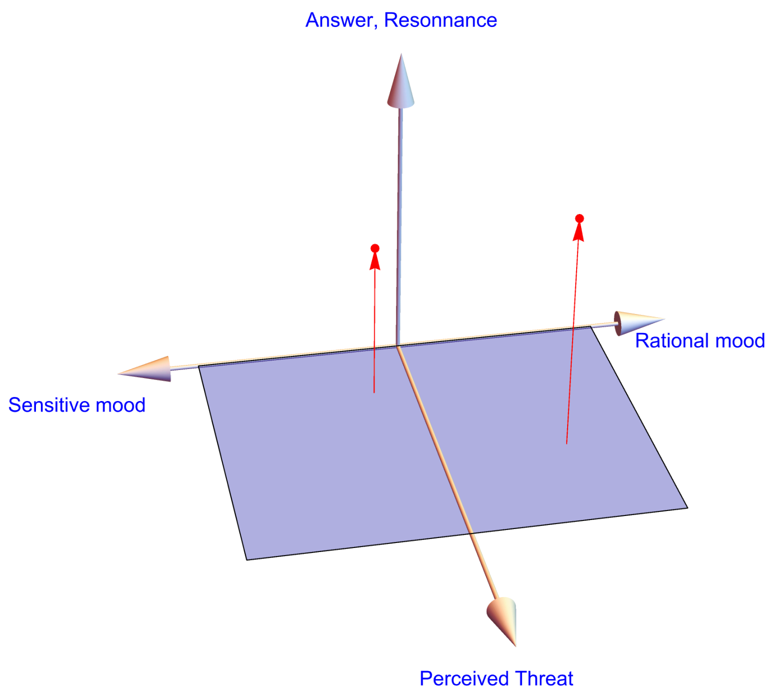


Figure 1: State Space. On the floor in blue the control space.

What does the state of opinion of a community look like? Figures 2 and 3 offer an answer. People's behavior is represented as a cloud of dots: one dot per person. Under the simplified hypothesis, where threat perception is the same for all people, the cloud lies in a vertical plane perpendicular to the threat axis. At low threat levels, the overall opinion of the community is fairly undifferentiated. The response of all members is largely homogeneous and it is possible to plot this opinion using a curve, as in Figure 2.

If the threat or solely its perception in the population is increased, the vertical plane moves accordingly. At the same time, the community splits into two camps, as also illustrated on Figure 3. One part of the people tends to perceive the threat seriously, while the other part has only a low response. It can even

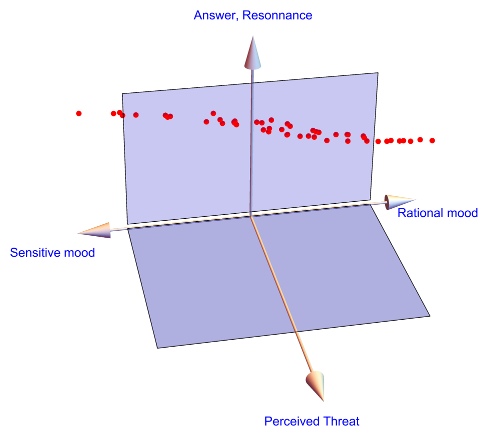


Figure 2: State Space by low perceived threat.

be observed that under the same attitudes -more or less sensitive or more or less rational - people are observed to have quite different resonances to the threat that is present. The community looks truly divided, or polarized. Is it possible to bind the two states together?

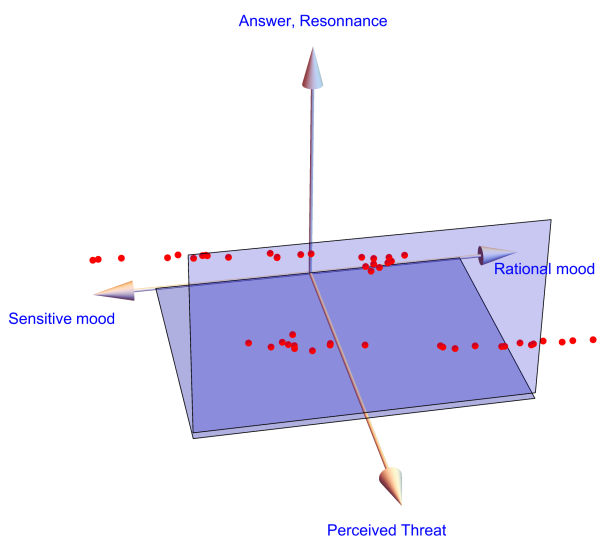


Figure 3: State Space by high perceived threat.

The response is positive, but in a surprising way, see Figure 4. The connecting surface, the so-called ***response surface***, can be seen as a function $z = f(x, y)$. It has a fold whose projection onto the underlying control plane determines two curves. These bifurcation sets divide the control space (threat setting) into two separate areas. It is easy to observe that the response surface within the branching set is ambiguous: above a point lying in this area, the response surface takes on three different values. In words, under the same threat and attitude, people show different opinions. More, on Figure 3 the central fold of the response surface is free of points. It is indeed possible to prove, even under general conditions, that the middle fold of the response surface is unstable and that the rest of the area is otherwise stable (See Appendix).

How should this be understood? Let us look at the point of a sensitive person under high perceived threat on Figure 4. Their point is on the upper left branch of the response surface: high resonance. If this person were to tone down their sensitivity and build up more rationality, their point would slide to the right on the surface (to the left on Figure 5) and eventually reach the knee of the surface. The point would not further slide on to the unstable middle fold, but would jump to the bottom fold. Thus, this person would abruptly change their mind and adopt a completely different view of the threat. Ergo, the branching set drawn in red on the control space in the fourth and fifth figures represents the location of these abrupt changes. This is why these curves are also described as branching or catastrophe sets. Finally, the convolution of the response surface, and above all the unstable property of the middle fold, explains why neutral people who show neither strong sensitivity nor cool rationality - they are located approximately at the zero point of the corresponding axis - can have radically different opinions.

Another geometric property of the fold can be observed on Figure 5 in a changed perspective: The right vertical red arrow on the picture maps the contact point of the two red curves (branching set) to the origin of the fold to the response surface. This point of contact (called the cusp point) is precisely punctual, unique, and appears early in the development of an increasing threat situation. However, since the response surface always remains smooth in this process, the emergence of the fold and the original branching cusp point associated to it is often not perceived. Especially in the context of social developments, the warning bells ring only later, when the split has unfolded.

It is also worth noting that an artifice of modern media rhetoric has developed, which consists in intensifying the participants' sense of threat in a debate. This creates a polarization, thanks to which only sharp and uncompromising statements are formulated. This procedure causes a narrowing of the perception corridor, which is usually desired, and enables the selection of those statements that are stamped as correct. This action can also be described geometrically by means of convolution.

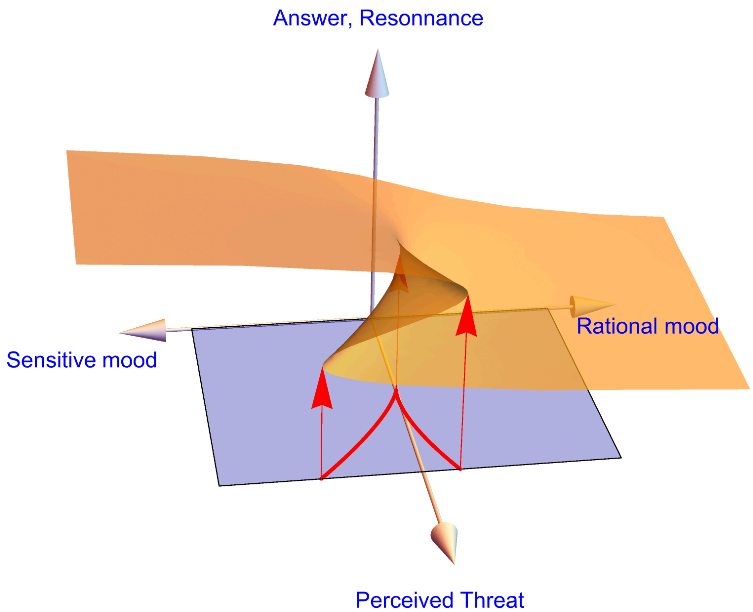


Figure 4: The crimp and the corresponding bifurcation set in the control space, on the floor.

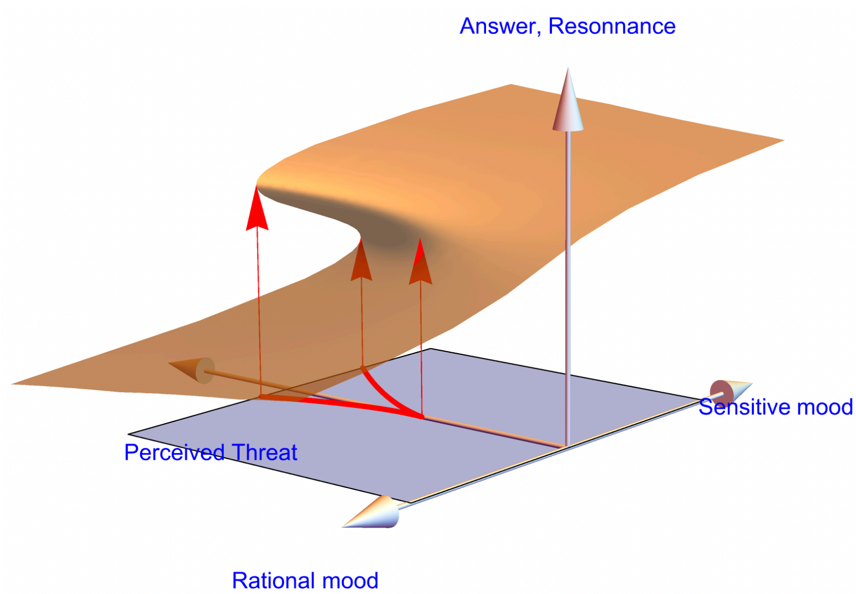


Figure 5: The crimp and the corresponding bifurcation set in another perspective.

Upshot

If the threat, or even just its perception, is increased in a community, then the community enters a polarized state. It is no longer tolerated to bear any fuzzy opinion about something and any change of opinion will occur as an abrupt, catastrophic sequence of events. This is a situation that naturally breeds conspiracy theories. In the Bible one even reads, under the Revelation of John, *"But because thou are lukewarm, and neither hot nor cold, I will spew thee out of my mouth"*.

Epilogue

To what extent is such an explanation to be trusted? This "catastrophic" worldview is not just an imaginative invention. It is based on a strictly proven theorem of mathematics. Within the framework of differential topology, René Thom (1923-2002, Fields Medal 1958) was able to prove that for four control variables and in a state space with an arbitrary number of dimensions, exactly seven different elementary catastrophes exist. Poetically, they were christened "the fold, the crimp, the swallowtail, the wave, the hair, the butterfly and the mushroom" (French: "le pli, la fronce, la queue d'aronde, la vague, le poil, le papillon et le champignon"). The elementary catastrophe presented in this paper is a crimp. Thom further speculated that these seven catastrophes could be regarded as letters and combined in a kind of universal grammar to elicit the variety of shapes in nature. Although the goals of this programme were ambitious, it must be acknowledged that certain successes have been achieved in the natural sciences: Phase transitions in physics or cellular differentiation in biology can be explained catastrophe-theoretically. Other applications have been tried in the fields of psychology, politics and even aesthetics. It has also been suggested that the "bull" and "bear" states of financial markets are subordinate to this catastrophic logic. In this respect, the feature shown in Figure 5 is generic. Even in more complex catastrophes, the transition to convolution occurs in a smooth, mostly unobtrusive manner. However, since physical-mathematical models are usually lacking, theory could only present qualitative representations. Nevertheless, Thom designed a "catastrophic grammar of semiotics" (sic !!!) and presented it in a later published book (Apologie du Logo, Apologetics of the Sign). He acknowledged that theory has only representational and no explanatory power and that its predictive capacity is limited in a fundamental way.

Finally, this essay makes no claim to be a new kind of meta conspiracy theory about conspiracy theories; it is merely a cool reflection from a forgotten mathematical- philosophical attempt of the past century.

References

1. Thom René. Modèles mathématiques de la morphogenèse. 1980. Christian Bourgeois Editeur

2. Thom René. Apologie du Logos. 1990. Hachette

Appendix: Few Computing

I wrote the programme that generated all this stuff years ago in Mathematica. The extract presented hereafter exhibits the kernel of the algorithm. The demanding graphics routines generating the figures in the main text are spared to the reader.

```
In[17]:= ClearAll[F, z, u, v];
```

According to René Thom's theory of catastrophes, the potential generating a crimp (F: un fronce, D: eine Kräuselfalte) is,

In[18]:= F[z_, u_, v_] = $\frac{z^4}{4} + \frac{u z^2}{2} + v z + 5$

Out[18]= $5 + v z + \frac{u z^2}{2} + \frac{z^4}{4}$

The gradient of this potential $\partial_z F_{(z,u,v)}$ is:

In[19]:= DFz[z_, u_, v_] = ∂_z F[z, u, v]

Out[19]= $v + u z + z^3$

The discriminant of the gradient $\Delta[\partial_z F_{(z,u,v)}]$ is

```
In[20]:= DiFz[u_, v_] = Discriminant[DFz[z, u, v] , z]
```

Out[20]= $-4 u^3 - 27 v^2$

rkorb The zeroes of the discriminant $\Delta[\partial_z F_{(z,u,v)}] = 0$ determine the geometric location forming the set of bifurcations generating the catastrophe

```
In[21]:= u /. Solve[Discriminant[DFz[z, u, v] , z] == 0, u]
```

Out[21]= $\left\{-3 \left(-\frac{1}{2}\right)^{2/3} v^{2/3}, -\frac{3 v^{2/3}}{2^{2/3}}, \frac{3 (-1)^{1/3} v^{2/3}}{2^{2/3}}\right\}$

The manipulate operator below shows the potential as a function of the control variables u and v. The multiple roots of the equation $\partial_z F_{(z,u,v)} = 0$ are displayed.

Two roots of the $\partial_z F_{(z,u,v)} = 0$ equation are conjugated complex when the discriminant is negative: $\Delta[\partial_z F_{(z,u,v)}] \leq 0$. In such a circumstance, the potential $F_{(z,u,v)}$ has only one global minimum and the catastrophe surface, unambiguous, is formed by one single foil.

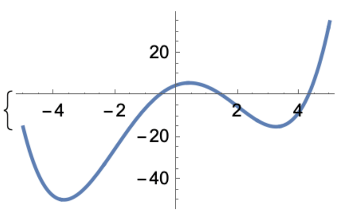
The bifurcation unfolds as soon as the discriminant becomes positive: $\Delta[\partial_z F_{(z,u,v)}] > 0$. The three roots of $\partial_z F_{(z,u,v)} = 0$ are then real. They correspond to the three equilibrium points of the potential and to the three foils of the catastrophe surface. The outer equilibrium points (located in the left and right valleys) are stable, with positive second derivatives of the potential. On the other hand, the negative second derivative at the central point (located on the top of the hill) corresponds to an unstable equilibrium, leading to the instability of the central foil of the catastrophe surface, as claimed in the body of the text.

In[34]:=
```
Manipulate[
 {Plot[F[z, u, v] , {z, -5, 5}],
  z /. Solve[DFz[z, u, v]  == 0, z]},
 {{u, -2}, -20, 20},
 {{v, 5}, -20, 20},
 SaveDefinitions → True
]
```

u

v

Out[34]=

{ 

, {-3.66916, 0.419316, 3.24984}}

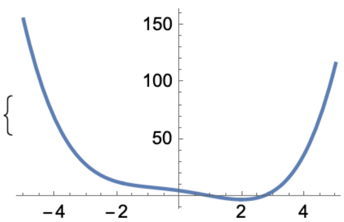
In[33]:=

```
Manipulate[
 {Plot[F[z, u, v] , {z, -5, 5}],
  z /. Solve[DFz[z, u, v]  == 0, z]},
 {{u, -2}, -20, 20},
 {{v, 5}, -20, 20},
 SaveDefinitions → True
]
```

Out[33]=

u

v

{ 

,

{-0.99497 - 0.984832 i, -0.99497 + 0.984832 i, 1.98994}}

The catastrophe surface is constructed in the space of three variables {v, u, Z} where v is substituted for z in the expression $\partial_z F_{(z,u,v)} = v + u\,z + z^3$, in such a way that the definition of Z is unambiguous in this projection. If this somewhat artificial condition is not met, the triple sheet cannot be processed correctly by Mathematica's graphical algorithm.

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```
In[23]:= DFu[u_, v_] = DFz[z, u, v] /. z → v
```

Out[23]= $v + u\,v + v^3$

Calculation of the geometric location of the bifurcation (into two branches), followed by graphical construction

```
In[24]:= CatEns[u_] = v /. Solve[∂v DFu[u, v] == 0, v] // FullSimplify
```

Out[24]= $\left\{-\frac{\sqrt{-1-u}}{\sqrt{3}}, \frac{\sqrt{-1-u}}{\sqrt{3}}\right\}$

```
In[25]:= Bifurq[u_] = DFu[u, v] /.
    v → {CatEns[u]⟦1⟧, CatEns[u]⟦2⟧} // FullSimplify
```

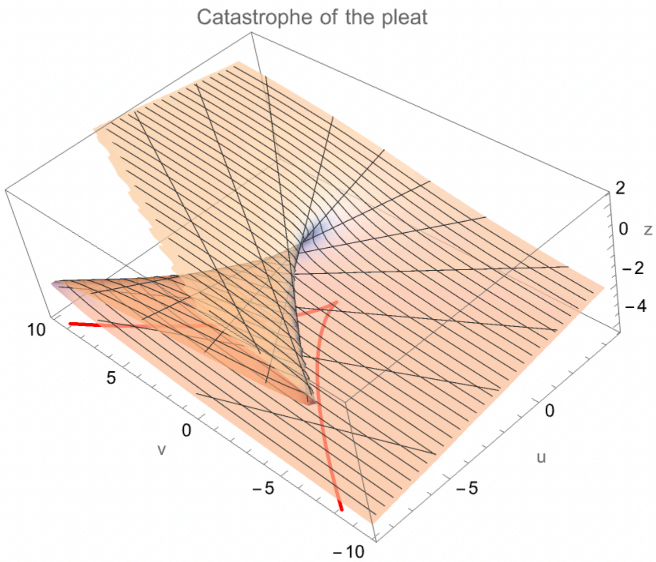
Out[25]= $\left\{\frac{2\,(-1-u)^{3/2}}{3\sqrt{3}}, -\frac{2\,(-1-u)^{3/2}}{3\sqrt{3}}\right\}$

```
In[38]:= Graphe0 = ParametricPlot3D[{ v, u, DFu[u, v]},
    {u, -10, 4}, {v, -8, 8},
    PlotStyle → {Opacity[0.7], LightPink},
    Mesh → 30];
Graphe1 =
  {ParametricPlot3D[{-4, u, Bifurq[u]⟦1⟧}, {u, -15, 15},
    PlotStyle → Red],
   ParametricPlot3D[{-4, u, Bifurq[u]⟦2⟧}, {u, -15, 15}, PlotStyle → Red]};
```

Final integration of both graphical objects

In[40]:=
```
Show[Graphe0, Graphe1,
 PlotLabel → "Catastrophe of the pleat",
 Axes → True, Boxed → True, Framed → True,
 AxesLabel → {"z", "u", "v"} ,
 PlotRange → {{-4.2, 2.2}, {-8.2, 4.2}, {-9.2, 9.2}}]
```

Out[40]=



The geometric location of the bifurcation (into two branches) can also be visualised as the intersection of the surface of the discriminant values $\Delta[\partial_z F_{(z,u,v)}]$ with the [u,v, 0] plane, both being computed as functions of u and v.

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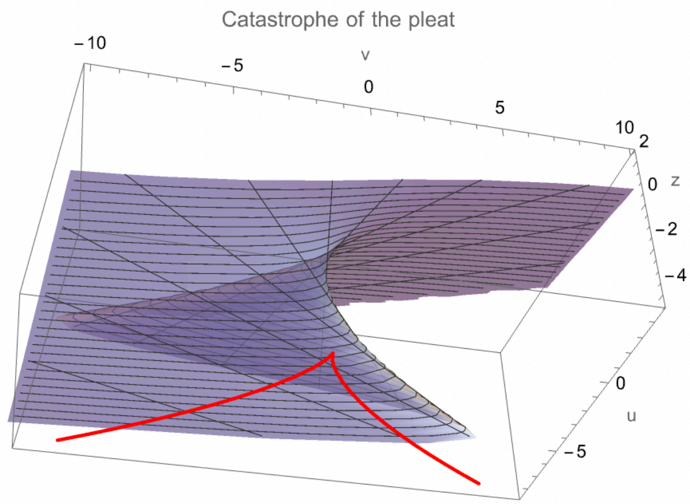
In[40]:= Show[Graphe0, Graphe1,
  PlotLabel → "Catastrophe of the pleat",
  Axes → True, Boxed → True, Framed → True,
  AxesLabel → {"z", "u", "v"} ,
  PlotRange → {{-4.2, 2.2}, {-8.2, 4.2}, {-9.2, 9.2}}]
```

Out[40]=



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